Astrophotography Formula

David N. Ault Jr.

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1 Signal To Noise Ratio

Howell, Koehn, Bowell, Hoffan Equation:

I = Photon Flux (photons/seconds)
η = Quantum Efficiency (e^-/photons)
τ = Exposure Time (seconds)
F = Number of Frames (no units)
n_s = Number of pixels in source aperture (no units)
n_B = Number of pixels in background aperture (no units)
N_d = Dark Current (e^-/seconds)
N_r = Read Noise (e^-)
G = Gain (e^-/ADU)
B_{sky} = Sky Background (ADU/second)
S/N = Signal to Noise Ratio (no units)

$$S/N = \frac{I\eta\tau\sqrt{F}}{\sqrt{I\eta\tau + n_s(1+\frac{n_s}{n_B})[B_{sky}G\tau + N_d\tau + N_r^2 + (0.289G)^2]}} \tag{1}$$

Howell, Koehn, Bowell, Hoffan modified for single pixel:

I = Photon Flux (photons/seconds)
η = Quantum Efficiency (e^-/photons)
τ = Exposure Time (seconds)
F = Number of Frames (no units)
N_d = Dark Current (e^-/seconds)
N_r = Read Noise (e^-)
G = Gain (e^-/ADU)
B_{sky} = Sky Background (ADU/second)
S/N = Signal to Noise Ratio (no units)

$$S/N = \frac{I\eta\tau\sqrt{F}}{\sqrt{I\eta\tau + B_{sky}G\tau + N_d\tau + N_r^2 + (0.289G)^2}} \tag{2}$$

Howell, Koehn, Bowell, Hoffan Equation (modified for subject S/N):

I = Photon Flux (photons/seconds)
η = Quantum Efficiency (e^-/photons)
τ = Exposure Time (seconds)
F = Number of Frames (no units)
n_s = Number of pixels in source aperture (no units)
n_B = Number of pixels in background aperture (no units)
N_d = Dark Current (e^-/seconds)
N_r = Read Noise (e^-)
G = Gain (e^-/ADU)
B_{sky} = Sky Background (ADU/second)
S/N = Signal to Noise Ratio (no units)

$$S/N = \frac{(I\eta\tau - B_{sky}G\tau)\sqrt{F}}{\sqrt{I\eta\tau + n_s(1 + \frac{n_s}{n_B})[B_{sky}G\tau + N_d\tau + N_r^2 + (0.289G)^2]}} \tag{3}$$

2 Ideal Exposure

P = Bits per pixel (no-units)
W = Full Well Depth (e^-))
N_r = Readout Noise (e^-))
τ = Test Exposure Time (seconds)
B = Measured Background Value (ADU)
b = Measured Bias Pedestal (ADU)
C = Contribution From Readout Noise $\{0.0 : 1.0\}$ (no-units)
G = Gain (e^-/ADU)
τ = Ideal Exposure Time (seconds)

If gain for your camera is unknown it can be estimated by: $G = \frac{W}{2^P}$

$$\tau = \frac{{N_r}^2}{[(C+1)^2 - 1](B-b)G} \tag{4}$$

3 Image Scale & Field of View

I_{scale} = Image Scale (arc-seconds / pixel) - The amount of sky one pixel covers
P_{size} = Pixel Size (μm)
F = Focal Length (mm)
B = Binning (no-units) - The binning mode 1 for 1x1, 2 for 2x2, etc.
C_x = Pixel count in X direction (no-units)
C_y = Pixel count in Y direction (no-units)
V_x = X Field of View (arc-minutes) - The amount of sky the sensor covers in the x direction
V_y = Y Field of View (arc-minutes) - The amount of sky the sensor covers in the y direction

$$I_{scale} = \frac{648P_{size}}{\pi FB}$$

$$V_x = \frac{C_x I_{scale}}{60}$$

$$V_y = \frac{C_y I_{scale}}{60}$$

4 Resolution

λ = Wavelength of Light (nm)
D = Diameter of Primary Mirror or Objective (mm)
Θ_r = Diffraction Limited Angular Resolution (radians)
Θ_s = Diffraction Limited Angular Resolution (arc-seconds)

$$\Theta_r = \frac{1.22\lambda}{1000000D} \tag{5}$$

$$\Theta_s = \frac{0.79056\lambda}{\pi D} \tag{6}$$

5 Polar Alignment

D = Star Drift (pixels)
I_{scale} = Image Scale (arc-seconds / pixel) - The amount of sky one pixel covers
τ = exposure time (seconds)
R_{drift} = Drift Rate (arc-seconds / second)
A_{dec} = Declination Angle (radians)
E = Drift Error (arc-seconds)

$$R_{drift} = \frac{DI_{scale}}{\tau} \tag{7}$$

$$E = \frac{300R_{drift}}{\pi \cos(A_{dec})} \tag{8}$$

6 Focal Reducer Effects

F_r = Focal Length of Reducer (mm)
D = Distance From Reducer to CCD (mm)
C = Compression Factor (no-units)
I_s = Image Circle of Scope or Reducer, which ever is smaller (mm)
- Sometimes Called Clear Aperture
I_c = Image Circle at CCD (mm)
F_s = Focal Length of Scope (mm)
F_e = Effective Focal Length (mm)

$$C = \frac{F_r - D}{F_r} \tag{9}$$

$$I_c = I_s C \tag{10}$$

$$F_e = F_s C \tag{11}$$

7 Critical Focus

λ = Wavelength of Light (nm)
R = Focal Ratio (no-units)
A_d = Airy Disk (mm)
C = Critical Focus Range (mm)

$$A_d = 0.00243932\lambda R \tag{12}$$

$$C = 0.00487864\lambda R^2 \tag{13}$$